

# ANALYSIS OF CASCADED QUASI-OPTICAL GRIDS

Scott C. Bundy, Wayne A. Shiroma, and Zoya B. Popović

University of Colorado  
Department of Electrical and Computer Engineering  
Boulder, Colorado 80309

**Abstract** — A general analysis method for cascaded grids is presented. The method is based on a full-wave theory for arbitrary periodic metal gratings printed on dielectrics and loaded with active or passive lumped devices. Each quasi-optical component is characterized by a multiport network, in which two of the ports represent the free-space regions on both sides of the grid surface, and the remaining ports are connected to the lumped loads. This approach allows cascading of quasi-optical components using transmission-line theory. Experimental validation between 2-18 GHz is presented for cascaded gratings loaded with lumped capacitors and resistors.

works can be cascaded in a transmission-line circuit, such as the one shown in Fig. 2. Using this building-block approach, an entire system composed of an arbitrary number of active and/or passive grids can be analyzed. Design iterations are fast, since changes in the dielectric constant or thickness are implemented by changing transmission-line characteristics.

## I Introduction

A number of active quasi-optical grids have been demonstrated in recent years: oscillators [1], amplifiers [2], mixers [3], phase shifters [4], [5], multipliers [6], and switches [7]. In active grids, solid-state devices periodically load a grating printed on a dielectric substrate. In some applications, devices may be loaded on both sides of the dielectric [8]. Two methods based on a unit-cell approach (infinite-grid approximation) have proven useful for designing active grids: an EMF theory (e.g., [1]) and a full-wave theory for grid oscillators [9]. Whereas the EMF method is limited to specific grid geometries where the current distribution can be assumed, the full-wave theory presented in [9] allows arbitrary periodic metal patterns on one or both sides of a dielectric substrate, and characterizes the passive part of the grid as an equivalent multiport network. This passive network is connected to an active device model, forming an equivalent circuit for the grid oscillator, which can then be analyzed using a conventional circuit simulator. This technique has been successfully demonstrated in several grid oscillator designs, but cannot be used for analyzing systems consisting of multiple quasi-optical components, such as the one shown in Fig. 1.

In this paper, we present an approach in which each planar structure in a cascaded system is characterized separately using the full-wave analysis. All of the equivalent net-

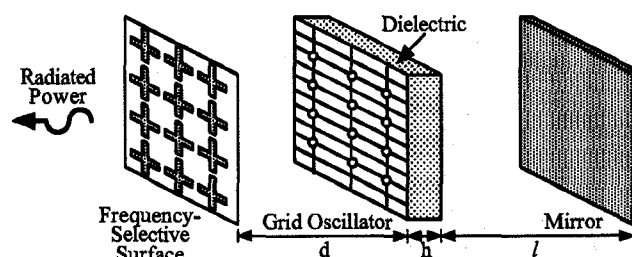


Fig. 1. An example of a simple quasi-optical system consisting of a frequency-selective surface cascaded with a transistor grid oscillator on a dielectric substrate. A mirror is located a distance  $l$  behind the dielectric.

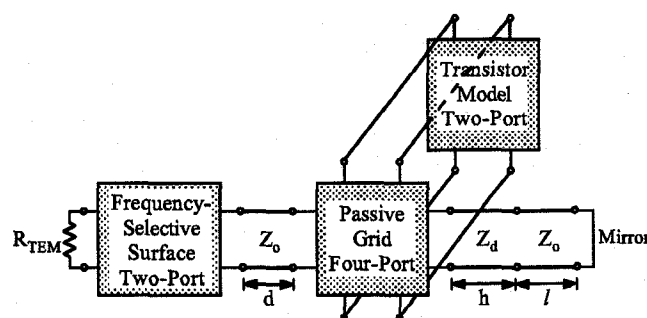


Fig. 2. Equivalent circuit for the quasi-optical system shown in Fig. 1. The passive grid geometry is represented by a four-port network, the frequency-selective surface is represented by a two-port network, the dielectric and free-space regions are modelled by TEM transmission lines, and the mirror is modelled by a short circuit. The resistor represents the radiation into free space.

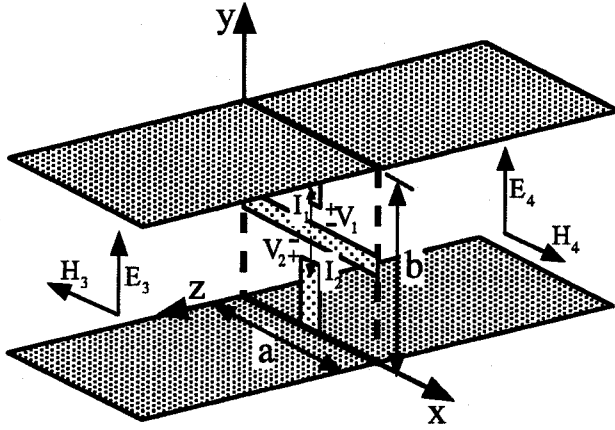


Fig. 3. The parallel-plate waveguide corresponding to a unit cell of width  $a$  and height  $b$ . This waveguide supports a TEM wave with a  $y$ -directed electric field and an  $x$ -directed magnetic field. When the unit cell at  $z=0$  is driven with a source, plane waves travel away from the structure in both the  $+z$  and  $-z$  directions.

## II Method of Approach

For the multiport characterization of the passive grid geometry presented in [9], the only accessible ports are those to which the active device is connected. In this work, we generalize the model by introducing two additional ports which represent the free-space regions on both sides of the grid. These two ports are equivalent TEM waveguides representing the unit-cell boundary conditions, as shown in Fig. 3. The solid lines represent the horizontal electric walls and the dashed lines represent the vertical magnetic walls. To characterize the grid as an equivalent multiport network, we define wave variables at each port. Wave variables at the device ports are defined in terms of the voltages and currents, while those at the TEM waveguide ports are defined in terms of the electric and magnetic fields. To find the  $s$ -parameters, we apply the following procedure. First, the device ports are driven with a voltage source while the waveguide ports are terminated in their matched impedance. This determines all  $s_{ij}$ , where  $j$  is a device port. Reciprocity is then used to find the  $s_{ji}$ 's. To determine the remaining  $s$ -parameters,  $s_{kl}$ , where both  $k$  and  $l$  are free-space ports, one of the waveguides is driven with an incident plane wave with the device ports shorted. All of the  $s$ -parameters are normalized to the TEM impedance  $\eta_0 b/a$ . We derived expressions for the  $s$ -parameters of completely passive structures, as well as for structures loaded with one-port or two-port devices. Details of the derivation can be found in [10].

The analysis above characterizes the grid metallization without including the dielectric substrate. A TEM transmission line connected to the multiport network is used to model the dielectric, but this neglects the added quasi-static capacitance seen at the device ports. To compensate for this,

a small lumped capacitor (a few fF) is placed across the device ports of the multiport. The capacitance value can be estimated from the geometry of the structure.

## III Experimental Results

To validate the theory, a number of grids were fabricated and measured; two examples are shown in Fig. 4. Each unit cell contains two ports which may be loaded with active or passive devices.

Measurements and simulations of the transmission coefficient of the unloaded grids are shown in Fig. 5. The bowtie and dipole grids exhibit resonances at 15.8 GHz and 10.2 GHz, respectively. Both resonances of the individual grids are observed in the frequency response of the cascaded system, as shown in Fig. 5(b). If the bowtie grid is loaded with a 0.5-pF capacitor in each unit cell, the resonance shifts from 15.8 GHz to 7.2 GHz, as shown in Fig. 6(a). Fig. 6(b) illustrates the effect of cascading the capacitively-loaded bowtie grid with the unloaded dipole grid. Once again, the resonances due to the individual grids are seen in the response of the cascaded system. When each unit cell of the bowtie grid is loaded with an additional 100- $\Omega$  resistor in the second port, the resulting characteristic is shown in Fig. 7(a). As expected, the resistor introduces loss and reduces the Q-factor of the grating. When this RC-loaded grid is cascaded with a capacitively-loaded dipole grid, the transmission characteristic of Fig. 7(b) is obtained.

Good agreement is observed between experiment and theory for both single and cascaded grids with different types of loadings. Although the examples presented here consider only passive loading, the theory is applicable to grids loaded with active devices as well. Using this approach, a system consisting of an arbitrary number of cascaded active and passive grids can be analyzed.

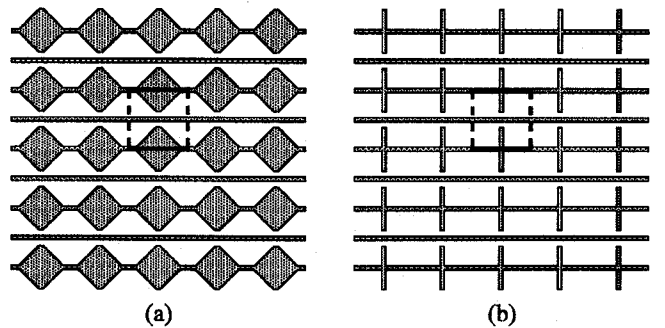
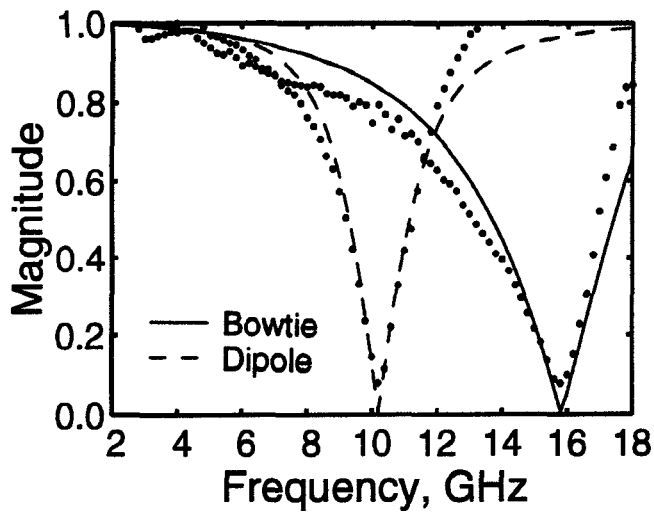
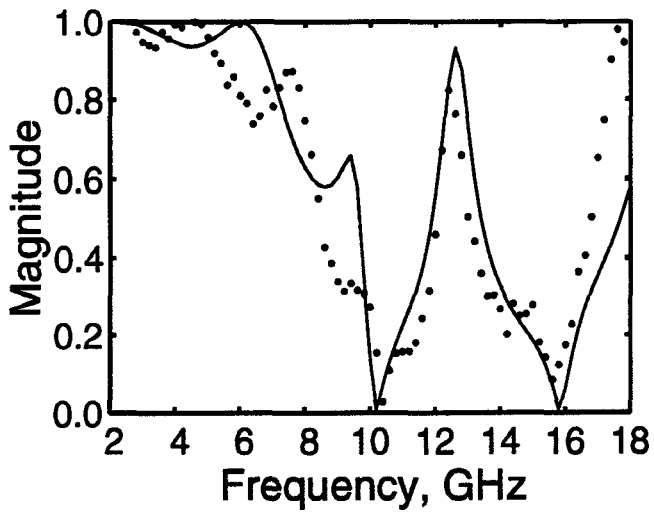


Fig. 4. Two different grid patterns, (a) one with bowtie radiating elements and (b) another with dipole radiating elements. Each unit cell contains two ports to which lumped elements may be connected. The grids are printed on 0.5-mm-thick *Duroid* substrates with  $\epsilon_r = 2.2$ .

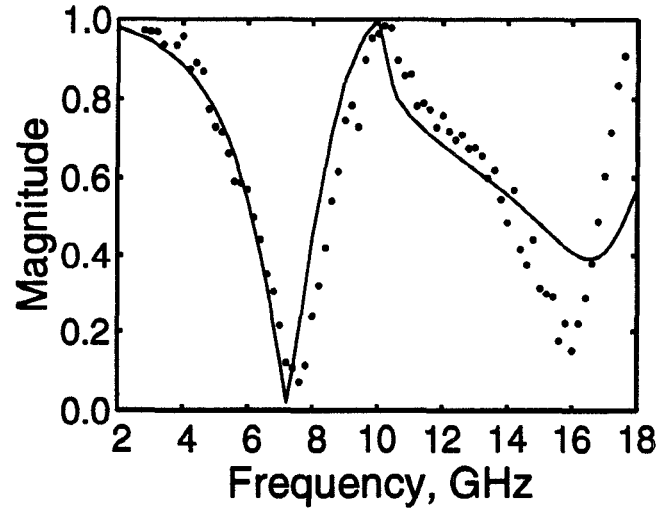


(a)

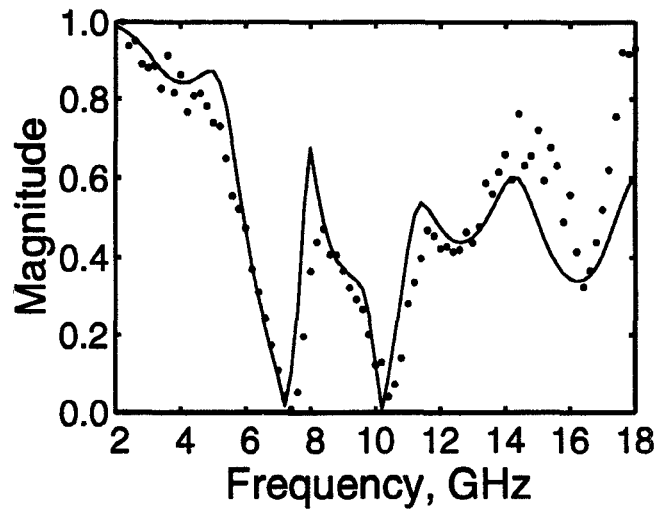


(b)

**Fig. 5.** Magnitude of the transmission coefficient of (a) each grid alone, and (b) the cascaded grids separated by 35 mm. The lines represent the theory and the dots represent the measured data.

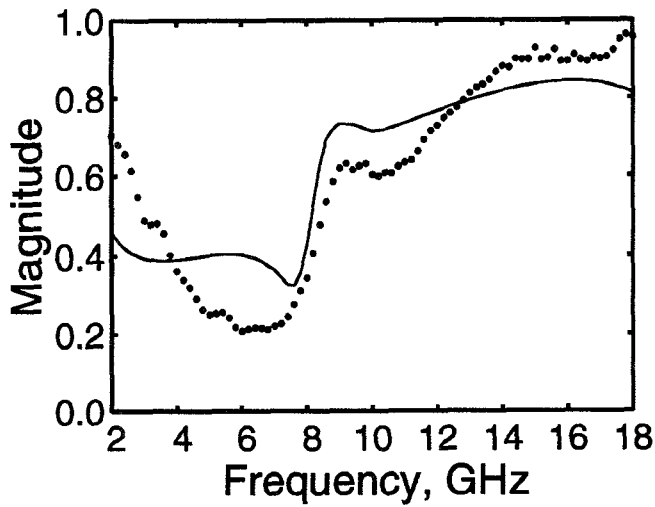


(a)

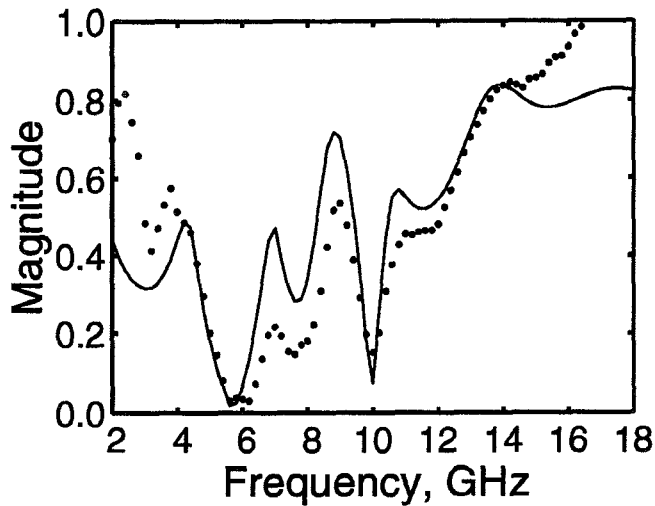


(b)

**Fig. 6.** Magnitude of the transmission coefficient of (a) the bowtie grid loaded with a 0.5-pF capacitor in each unit cell, and (b) the capacitively-loaded bowtie grid cascaded with the unloaded dipole grid 40 mm away. The solid line represents the theory and the dots represent the measured data.



(a)



(b)

Fig. 7. Magnitude of the transmission coefficient of (a) the bowtie grid loaded with a 0.5-pF capacitor and 100- $\Omega$  resistor, and (b) the RC-loaded bowtie grid cascaded with a capacitively-loaded dipole grid 40 mm away. The solid line represents the theory and the dots represent the measured data.

## Acknowledgment

This work was supported in part by the U. S. Army Research Office under contract #DAAL03-92-G-0265 and the AASERT program, and in part by NSF under a Presidential Faculty Fellow Award.

## References

- [1] Z. B. Popović, R. M. Weikle II, M. Kim, and D. B. Rutledge, "A 100-MESFET planar grid oscillator," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 193-200, Feb. 1991.
- [2] M. Kim, E. A. Sovero, J. B. Hacker, M. P. De Lisio, J.-C. Chiao, S.-J. Li, D. R. Gagnon, J. J. Rosenberg, and D. B. Rutledge, "A 100-element HBT grid amplifier," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 1762-1771, Oct. 1993.
- [3] J. B. Hacker, R. M. Weikle II, M. Kim, M. P. De Lisio, and D. B. Rutledge, "A 100-element planar Schottky diode grid mixer," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 557-562, Mar. 1992.
- [4] W. W. Lam, C. F. Jou, H. Z. Chen, K. S. Stolt, N. C. Luhmann Jr., and D. B. Rutledge, "Millimeter-wave diode-grid phase shifters," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 902-907, May 1988.
- [5] L. B. Sjogren, H.-X. Liu, X. Qin, C. W. Domier, and N. C. Luhmann Jr., "Phased-array operation of a diode grid impedance surface," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 565-572, Apr. 1994.
- [6] C. F. Jou, W. W. Lam, H. Z. Chen, K. S. Stolt, N. C. Luhmann Jr., and D. B. Rutledge, "Millimeter-wave diode-grid frequency doubler," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 1507-1514, Nov. 1988.
- [7] K. D. Stephan and P. F. Goldsmith, "W-band quasi-optical integrated PIN diode switch," *1992 IEEE MTT-S Symp. Dig. (Albuquerque, NM)*, pp. 591-594.
- [8] T. B. Mader, S. C. Bundy, and Z. B. Popović, "Quasi-optical VCOs," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 1775-1781, Oct. 1993.
- [9] S. C. Bundy and Z. B. Popović, "A generalized analysis for grid oscillator design," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 2486-2491, Dec. 1994.
- [10] S. C. Bundy, *Analysis and Design of Grid Oscillators*, Ph.D. Dissertation, University of Colorado, Boulder, Dec. 1994.